***Calculus 12 Learning Objectives***

1. **Explore the concepts of average and instantaneous rate of change.**

* Determine the ***average rate of change*** (***slope of a secant line***) of a function over an interval.
* Demonstrate an understanding that the ***instantaneous rate of change*** of a function at a point (***slope of a tangent line***) is the limiting value of a sequence of average rates of change.
* Determine the slope of a curve at a given point.
* Determine whether a curve has a ***tangent line*** at a given point.
* Determine the equation of a tangent line to a curve at a given point.
* Determine the equation of a ***normal line*** to a curve at a given point.
* Calculate and interpret average rates of change drawn from a variety of applications.
* Solve problems involving instantaneous rates of change drawn from a variety of applications.

1. **Determine the derivative of a function by applying the definition of derivative.**

* Determine the ***derivative*** of a function, ***f(x)*** by using the limit definition of the derivative,

or

* Define and evaluate the derivative at ***x = a***.
* Use alternate notation interchangeably to express derivatives (*i.e.,* ***f’(x), , y’****, etc*.*).*
* Given the graph of the derivative of a function, sketch a graph of the function.
* Given the graph of a function, sketch a graph of its derivative.
* Determine whether a function is differentiable at a given point.
* Explain why a function is not differentiable at a given point, and distinguish among corners, cusps, discontinuities, and vertical tangents. Determine all values for which a function is differentiable.

1. **Apply derivative rules to determine the derivative of a function, including: Constant Rule; Power Rule; Constant Multiple Rule; Sum Rule; Difference Rule; Product Rule; Quotient Rule.**

* Determine the derivatives of functions, using the ***Constant, Power, Constant Multiple, Sum, Difference, Product, and Quotient Rules***.
* Determine second and higher-order derivatives of functions.
* Solve problems involving derivatives drawn from a variety of applications (including applications in the social and natural sciences).

1. **Find derivatives of trigonometric functions.**

* Establish each of the following trigonometric limits, using informal methods:

* Derive the derivatives of the six basic trigonometric functions.
* Determine the derivative of a trigonometric function.
* Use the ***Sandwich Theorem (Squeeze Theorem)*** to find certain limits indirectly.
* Solve a problem involving the derivative of a trigonometric function.
* Evaluate limits involving trigonometric functions using ***L’Hôpital’s Rule***:

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1. **Apply the Chain Rule to determine the derivative of a function.**

* Demonstrate an understanding of the ***Chain Rule***.
* Determine the derivative of a ***composite function***, using the Chain Rule.
* Solve a problem involving the derivative of a composite function.
* Determine the derivative of a relation, using ***implicit differentiation***.
* Determine the equation of the tangent and normal lines to the graph of a relation at a given point.
* Determine the second derivative of a relation, using implicit differentiation.
* Solve problems involving implicit differentiation drawn from a variety of applications.

1. **Solve problems involving inverse trigonometric functions.**

* Explain the relationship between the primary trigonometric functions and the corresponding ***inverse trigonometric*** function.
* Explain why inverse trigonometric functions have restricted domains and ranges.
* Determine the exact value of an expression involving an inverse trigonometric function.
* Simplify an expression involving an inverse trigonometric function.
* Determine the domain of an inverse trigonometric function.
* Sketch the graph of an inverse trigonometric function.
* Determine the derivative of inverse trigonometric functions including composites.
* Solve problems involving the derivative of an inverse trigonometric function.

1. **Find limits and derivatives of exponential and logarithmic functions.**

* Establish the exponential limit using informal methods.
* Determine the derivatives of the exponential functions ***y = ax*** and ***y = ex***, and of the logarithmic functions ***y = logax*** and ***y = lnx***.
* Determine the derivative of composite exponential and logarithmic functions.
* Determine the derivative of a function using logarithmic differentiation.
* Solve a problem involving the derivative of an exponential or a logarithmic function. (e.g. exponential growth and decay problems).
* Evaluate limits involving logarithmic functions using L’Hôpital’s Rule.

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1. **Use calculus techniques to sketch the graph of a function.**

* Determine the ***local*** and ***global (absolute) extreme values*** of a function.
* Demonstrate an understanding of the ***Extreme Value Theorem***.
* Determine the ***critical*** and ***stationary*** points of a function.
* Demonstrate an understanding of the ***Mean Value Theorem***.
* Determine the intervals on which a function is increasing and decreasing.
* Use the ***First Derivative Test***to locate the ***local extrema*** of a function.
* Use the ***First or Second Derivative Test***to classify the local extrema of a function.
* Use the ***Concavity Test (second derivative)*** to determine the ***intervals of concavity*** of a function.
* Determine the ***points of inflection*** of a function.
* Determine ***asymptotic*** and ***end behaviour*** of a function using limits.
* Determine the key features of the graph of a function, using the techniques of differential calculus, and use these features to sketch the graph without technology.
* Sketch the graph of a function, using information about its derivative.

1. **Use calculus techniques to solve optimization problems.**

* Determine the equation of the ***objective function*** to be optimized in an optimization problem.
* Determine the equations of any parameters necessary in an optimization problem.
* Solve an optimization problem drawn from a variety of applications, using calculus techniques.
* Interpret the solution(s) to an optimization problem.

1. **Use linearization (and Newton’s Method - optional) to solve problems.**

* Use ***linearization*** to approximate a numerical expression.
* Solve linearization problems drawn from a variety of applications.
* **Optiona:l** Determine the differential of a function.
* **Optional:** Use Newton’s Method to approximate the solution(s) of an equation.

1. **Solve problems involving related rates.**

* Develop a mathematical model for a ***related rates*** problem.
* Solve problems involving related rates, drawn from a variety of applications.
* Interpret the solution to a related rates problem.

1. **Determine the definite integral of a function.**

* Estimate an area using a finite sum.
* Explore ***left, right and midpoint rectangular approximations***.
* Relate a ***Riemann sum*** to a definite integral.
* Evaluate a ***definite integral*** using an area formula or Riemann sums to solve problems.

1. **Determine the antiderivative of a function.**

* Explain the meaning of the phrase ***“F(x) is an antiderivative of f(x)”****.*
* Determine the general antiderivative of functions.
* Use antiderivatives notation appropriately *(i.e.,* ***F(x) = ∫f(x)*** for the antiderivative of ***F(x)***)*.*
* Identify the properties of ***antidifferentiation***.
* Determine the antiderivative of a function given initial conditions.
* Understand the relationship between the derivative and definite integral as expressed in ***Part I***of the ***Fundamental Theorem of Calculus***.
* Evaluate definite integrals from antiderivatives using ***Part II***of the *Fundamental Theorem of Calculus*.
* Calculate the definite integral of a function over a closed interval ***[a,b].***
* Use substitution to determine the indefinite or definite integral of a function.

1. **Solve problems that involve the application of the integral of a function from a variety of fields, including the physical and biological Sciences, economics and business**.

* Use a definite integral to determine the area under a function, and above the x-axis, from ***x = a*** to ***x = b***.
* Determine the area between two functions.
* Use integration to solve problems about motion of a particle along a line that involves:

- computing the displacement given the initial position and velocity as a function of time

- computing velocity and/or displacement given the suitable initial conditions and acceleration as a function of time.

* Use integration to solve problems from biological sciences, economics and business.